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# Perspectives on Jet Noise

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## Introduction

A THIRD of the Dryden lectures so far have dealt with some aspect of turbulent flow. This was a subject of considerable interest to Hugh Dryden, and to which he contributed significantly with several authoritative reviews. Jet noise, the subject of the present lecture, is a byproduct of turbulence. For the most part, the edifice of jet noise theory has sat atop the structure of turbulence theory, adding nothing: the turbulence was assumed to be known statistically, and the jet noise computed therefrom. But in the last half decade many researchers have been taking a more integrated view, stimulated by new knowledge of the behavior of instability waves and of vortices. This will form a part of our review.

As a further perspective, it is apparent that aeroacoustics is a small field of overlap between aeronautics and acoustics. Jet noise is a still smaller, highly specialized, subarea. Thus, in addressing jet noise for this lecture—particularly theoretical aspects—I have been faced with the dilemma of how to reach a general aerospace audience. As it happens, an early view of jet noise generation—circa the sixties, but still defensible—is very graphic and easy to visualize: this *simple view* has been chosen for the first part. An aerodynamicist should be able to come away with a good conception of the basic mechanisms.

As for the last part, the ultimate target of jet noise research has been *suppression*. Quite a number of approaches have been tried, many of them ad hoc. It happens that these schemes, for the most part, are far easier to portray and comprehend than the theories of jet noise. This suggests their choice for the final section. Again, an aerodynamicist should come away with a clear conception.

This leaves the middle section. I will let this take the form of a guided tour through some of the many *further developments* in jet noise theory. There is no escape from the specialized nature of the material; however, I will make every effort to strip away the concealment of jargon and aim at simple physical interpretations.

A major further objective has been unification. Many of these further developments seem divergent. The theorists are divided into different camps, different schools of thought. I have attempted to bridge this divergence, to find the equivalences, the common ground. This effort has required a lot of spadework, but the greatest divergence appears to have been largely resolved. I refer to the split between the Lighthill and Lilley theories: between two different roles for the mean-flow shear. The thesis will be made herein, with evidence presented, that they are not so different after all.

## The Simple View

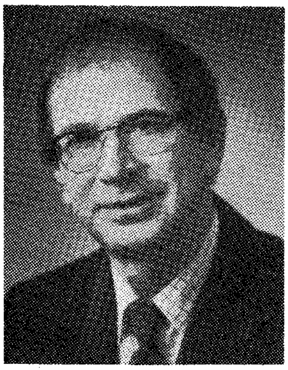
### Generation Process

A turbulent flow contains an apparently random pattern of momentum, vorticity, and pressure. These variables are interconnected by the governing dynamic and continuity equations. Any one of the three[1-9] can serve as the vehicle for predicting noise generation. Initially, we choose a pressure formulation as the simplest (*dilatation theory*).

The pressure varies from point to point throughout the flow in a seemingly chaotic fashion. We may crudely model a pressure "high" as the stagnation pressure resulting from colliding filaments of flow. Such filaments—we might call them "jetlets"—have stochastic orientations and locations (Fig. 1). Where a pair collide, the stagnation region is compressed slightly; but the compression is transient, owing to the flow unsteadiness. This unsteady *dilatation* of fluid elements, driven by inertial (momentum) effects, generates sound.[6-9]

### Synthesis of Pressure Pattern

If we oversimplify, the radiated sound is predicted to have a certain quasielliptic directional pattern (intensity vs direction) that is designated *basic* in Fig. 2. This scenario, however, ignores *convection* of the sources by the flow; the associated wave crowding strongly enhances the sound intensity in the



**Professor H. S. Ribner obtained his B.S. from Caltech and his M.S. and Ph.D. (1939) from Washington University (St. Louis), all in physics. After a short period in gravity meter development for oil exploration, he spent eight years at the NACA Langley Laboratory and five years at the Lewis Laboratory, variously as aerodynamicist and section head (Stability Analysis Section, Langley, Boundary Layer Sections, Lewis). He joined the staff of the University of Toronto Institute for Aerospace Studies in 1955, was a Visiting Professor at University of Southampton in 1960-61, and a Staff Scientist at NASA Langley in 1975-76. His research interests have covered, somewhat chronologically, X-rays, cosmic rays, gravimetry, aerodynamics, aeroacoustics, sonic boom, and the acoustics of thunder. Professor Ribner is a Fellow of AIAA, the American Physical Society, the Acoustical Society of America, the Canadian Aeronautics and Space Institute (CASI), and the Royal Society of Canada. He was the CASI Turnbull Lecturer in 1968, Chairman of the International Civil Aviation Organization Sonic Boom Panel in 1970-71, and received the AIAA Aeroacoustics Award in 1976. Currently, he divides his time between the University of Toronto and NASA Langley Research Center.**

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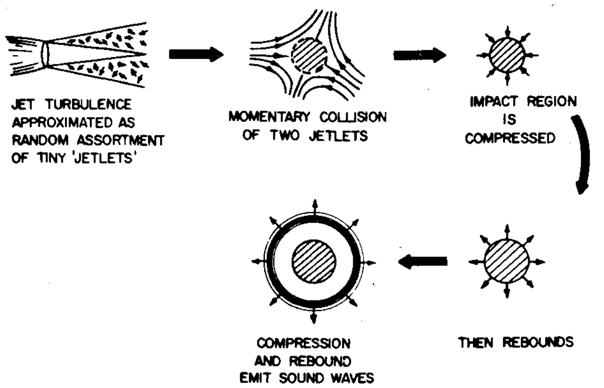



**Fig. 1 Generation of jet noise according to simplified dilatation theory.**

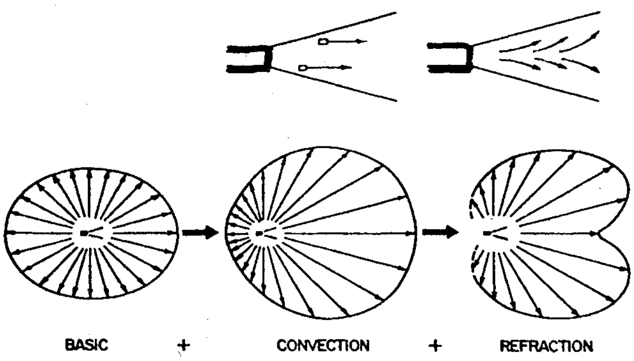



**Fig. 2 Convection and refraction modify basic pattern of intensity vs direction.**

downstream direction (Fig. 2, middle sketch). Also ignored have been velocity gradients (and temperature gradients in hot jets) that turn the sound rays outward. This *refraction* weakens the intensity in a range of directions near the axis to yield the dimple in the final pattern ("refraction valley" or "cone of relative silence"). In the following sections we shall indicate how the *basic* pattern comes about and elaborate on the effects of *convection* and *refraction.*

## Governing Equations

Mathematically, referring to any small volume within the flow, the dilatation concept gives, as an approximation,

$$\frac{\partial\,(\text{volume})}{\partial t} \sim -\frac{\partial\,(\text{pressure})}{\partial t}$$

$$\frac{\partial^2\,(\text{volume})}{\partial t^2} \sim -\frac{\partial^2\,(\text{pressure})}{\partial t^2} \sim \frac{\text{source strength}}{\text{unit volume}} \tag{1}$$

The second time derivative figures here because it is the radial *acceleration* of the little sphere that is effective in generating sound. The proportionality factor may be taken for our purposes as (ambient sound speed)$^{-2} = c_0^{-2}$.

This effective acoustic source strength leads to the sound perturbation pressure (deviation from ambient) radiated to a point at a large distance $x$ (the "far field") as[6-9]

$$p(x,t) = \frac{1}{4\pi x}\int_V \left[-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\,(\text{local pressure})\right]\mathrm{d}V \tag{2}$$

dilatation theory

taken over the jet volume $V$ at a time, for each $\mathrm{d}V$, earlier than $t$ by the acoustic travel time from $\mathrm{d}V$ to $x$ at speed $c_0$.

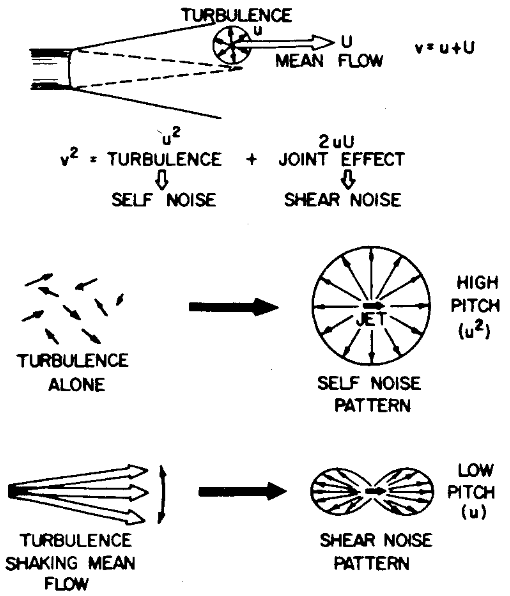



**Fig. 3 Components of basic pattern of intensity vs direction, with interpretation.**

Strictly speaking, the pressure is not exactly the local pressure: it differs by a contribution due to sound waves passing through (i.e., not generated within) $\mathrm{d}V$ (Ref. 9). However, the difference between this "pseudosound" pressure and the full local pressure is probably immeasurably small.

An equivalent equation in terms of *momentum flux* (Reynolds stress) can be obtained from Eq. (2) by means of a momentum balance utilizing a control surface.[9] The result is

$$p(x,t) = \frac{1}{4\pi x}\int_V \left[\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\,(\rho v_x^2)\right]\mathrm{d}V \tag{3}$$

Lighthill – Proudman theory

where $v_x$ is the velocity component in the direction of the field point $x$; this may also be written

$$v_x = v_j x_j / x \tag{4}$$

to be summed over the repeated index $j$. Insertion of Eq. (4) into Eq. (3) gives a form originally obtained by Lighthill[1,2] directly from the mass and momentum conservation equations. Equation (3) is a simpler variant due to Proudman.[10]

The integrands of Eqs. (2) and (3), and hence $p$, can be known only in a statistical sense: e.g., as mean squares. We gloss over messy details (cf. Refs. 1, 2, or 11) by writing

$$\overline{p^2}\,(x) \sim \frac{1}{x^2 c_0^2}\int_V \overline{\left(\frac{\partial^2 (\rho v_x^2)}{\partial t^2}\right)^2} L^3 \mathrm{d}V \tag{5}$$

where $L^3$ is a "correlation volume" (or, more vaguely, "eddy volume") within which $\rho v_x^2$ is effectively uniform or coherent. The integrand is a function of observer direction $x$ as well as position of $\mathrm{d}V$ in the jet.

The velocity $\boldsymbol{v}$ is compounded of turbulence $\boldsymbol{u}$ plus mean flow $U$. Thus the component in the direction of the observer $\boldsymbol{x}$ is $v_x = U_x + u_x$. With the usual approximation of $\rho$ by the ambient value $\rho_0$, this gives[11]

$$\rho_0 v_x^2 = \underset{\text{self}}{\rho_0 u_x^2} + \underset{\text{shear}}{2\rho_0 u_x U_x} + \underset{\text{const}}{\rho_0 U_x^2} \tag{6}$$

The first term is responsible for the "self" noise due to turbulence alone. The second represents a joint contribution of turbulence and mean flow: it is called "shear" noise because the volume integral of this term vanishes if the mean flow shear vanishes. The last term has usually been dropped because of the constancy of $U_x$, ignoring the variability of $\rho$; new analysis shows it can contribute appreciably to the noise at the higher jet velocities [term in Eq. (25) with factor $p'/\rho_0 c_0^2$].

#### Basic Pattern

Figure 3 gives a schematic interpretation of the self noise and shear noise source terms. The accompanying directional intensity patterns were worked out from Eq. (5) with $L^3$ functionally defined. The *figure 8* or dipole-like directional pattern for the shear noise should be particularly noted for later comparison with experiment. The combined self-noise, shear-noise pattern is (Ref. 11, corrected in Ref. 12),

$$\overline{p^2} = \underset{\text{self noise}}{A} + \underset{\text{shear noise}}{B[(\cos^2\theta + \cos^4\theta)/2]} \tag{7}$$

This describes the quasielliptic basic pattern referred to earlier.

Note further that if $u_x$ is sinusoid, $\sim e^{i\omega t}$, then $u_x^2 \sim e^{i2\omega t}$, a frequency doubling. This suggests that *the self-noise spectrum lies roughly an octave above the shear-noise spectrum*. This prediction is tested later in the paper.

#### Convection

Proper evaluation of Eq. (5) requires allowance for convection of the sources by the flow. Under certain assumptions concerning the source field statistics, one obtains[9,13] a *convection factor*

$$C^{-5} = [(1 - M_c \cos\theta)^2 + \alpha^2 M_c^2]^{-5/2} \tag{8}$$

that multiplies the basic directivity, Eq. (7). This enhances the downstream emission as shown in the center sketch of Fig. 2 ($\alpha$ is a pure number of order 0.3-0.55). A computer movie on convection effects accompanies the lecture.

#### Refraction

In principle, the Lighthill theory allows for refraction of sound by the flow. In practice, the replacement of the slightly variable density $\rho$ by the ambient value $\rho_0$ suppresses the refraction. Therefore, resort has been made to experiment[14,15] to quantify the effect. The injection of a pure tone into a jet via a "point" source is the chosen technique: the deviation of the intensity pattern from circular measures the degree of refraction. In Fig. 4, the pure tone "refraction valley" closely matches that of the narrow-band filtered jet noise, even for a cryogenic cold jet. A theory specifically for refraction[16,17] shows satisfactory agreement.

## Further Developments

#### Lighthill Equation as Starting Point

We now give a more rigorous derivation than the arguments leading from Eq. (1) to Eq. (4); it yields an extra source term in the Lighthill equation, often neglected. To a sufficient accuracy for our purposes the fluid may be treated as inviscid and non-heat-conducting (hence homentropic). The conservation equations then read

Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0 \quad i = 1,2,3 \tag{9}$$

Momentum

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i v_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} \tag{10}$$

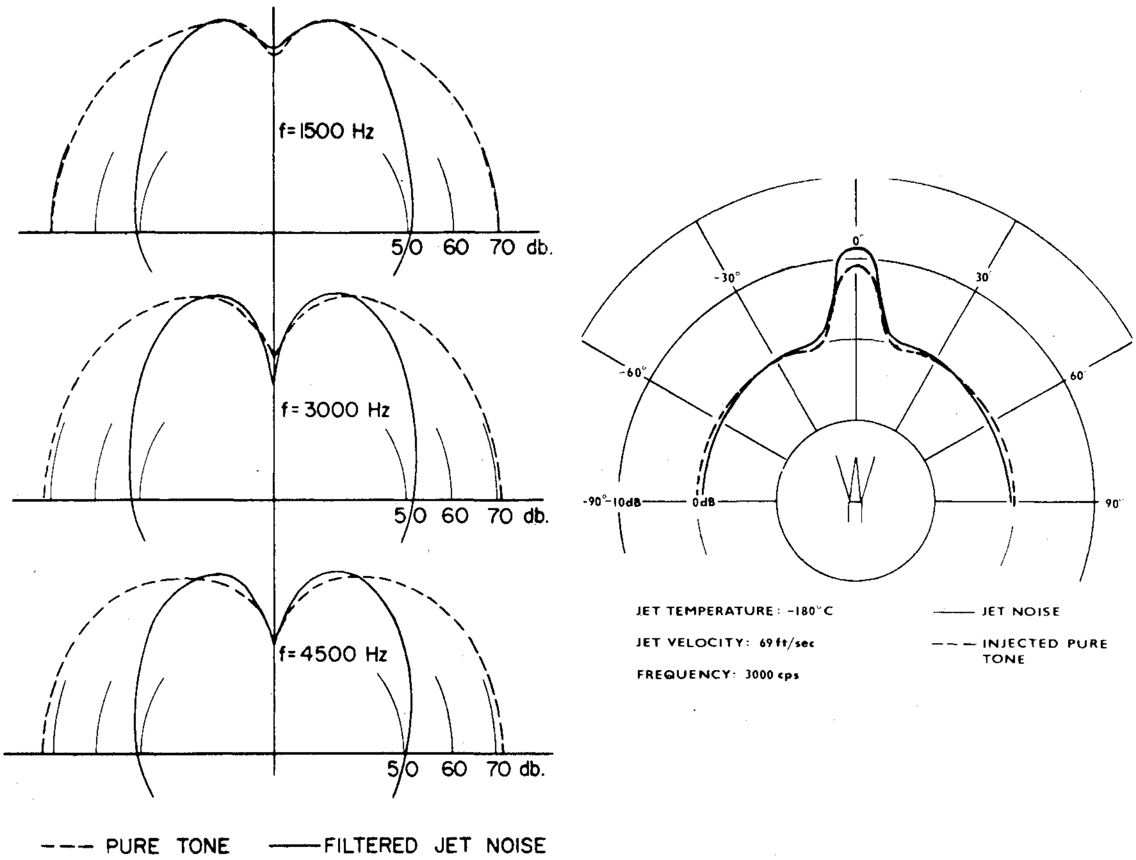



Fig. 4 Match of noise intensity patterns to indicate refraction-dominated zone. Left-hand side: room temperature jet with axial dimple (rays turn outward); right-hand side: very cold jet (−180°C) with focused lobe (rays turn inward).

Apply $\partial/\partial t$ to Eq. (9), $\partial/\partial x_i$ to Eq. (10), and subtract

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 \rho v_i v_j}{\partial x_i \partial x_j} = \frac{\partial^2 p}{\partial x_i^2} \equiv \nabla^2 p$$

Addition of $c_0^{-2}\partial^2 p/\partial t^2$ to both sides and rearrangement then yields the full *Lighthill equation*[1] restated in terms of pressure:

$$\underbrace{\left[\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]}_{\text{wave operator}} p = \underbrace{\frac{\partial^2 \rho v_i v_j}{\partial x_i \partial x_j} - \frac{\partial^2}{\partial t^2}\left(\rho - \frac{p}{c_0^2}\right)}_{\text{sources}} \equiv q \tag{11}$$

Outside the jet flow the terms on the right-hand side lumped together as $q$ vanish to first order, and Eq. (11) is merely a homogeneous wave equation: it describes the propagation of sound in a source-free medium. Within the jet $v_i$, $v_j$ are non-negligible turbulent flow velocity components, and the terms of $q$ are no longer negligible: Lighthill interpreted $q$ as a spatial distribution of acoustic sources generating the jet noise. He showed further how this $q$, neglecting the second term, leads to the result referred to just after Eq. (4).

At a large distance $\boldsymbol{x}$ (far-field) the solution of Eq. (11) is

$$p(x,t) = \frac{1}{4\pi x}\int_V q(y,t')\,\mathrm{d}^3 y \tag{12}$$

which is of the form Eq. (3) with $q$ as the integrand. Here, as before, $t'$ is retarded relative to $t$ by the travel time from source point $\boldsymbol{y}$ to field point $\boldsymbol{x}$ at the sound speed $c_0$.

The integrand of Eq. (12) is a "snapshot" of the spatial distribution of $q$ in retarded time $t'$. This stochastic pattern is neither measurable nor predictable in any practical sense. However, we are normally interested in statistical quantities such as mean square pressure and its power spectrum, rather than the instantaneous pressure $p(x,t)$. For this purpose, the snapshot of $q$ is replaced by a measurable statistical quantity: the correlation of $q$ at two points and two times. The integral of this correlation, normalized to unit peak value, over a volume encompassing the jet, is the correlation volume $L^3$ of Eq. (5). Corresponding to Eq. (5), Eq. (12) goes over to

$$\overline{p^2}(x) = \frac{1}{16\pi^2 x^2}\int_V \overline{q^2}(y)\,L^3(y;x/x)\,\mathrm{d}^3 y \tag{13}$$

plus an associated integral for the evaluation of $L^3$.

The essence of Lighthill's idea,[1,2] then, is that the sources $q$ in Eq. (12) are *known* or *modeled* (statistically in terms of correlations) from our knowledge of jet turbulence. Plugging the modeled $q$ (correlations) into Eq. (13) and its companion for $L^3$ then leads to prediction of the noise pattern $\overline{p^2}(x)$. For less quantitative *scaling laws*, however, $\overline{q^2}$ and $L^3$ merely may be estimated and used in Eq. (13).

This modeling approach of Lighthill has been very fruitful: it has delineated quantitatively the properties of jet noise discussed earlier, with one exception, the refraction effect. This shortcoming is traceable to the treatment of the density $\rho$ in the source terms $q$ of Eqs. (11-13). By the gas law for homentropic flow,

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = \frac{1}{c^2}\frac{\mathrm{D}p}{\mathrm{D}t} \tag{14}$$

for a moving fluid element. Thus $\rho$ contains perturbations $\mathrm{d}\rho$ that depend on the pressure perturbations $\mathrm{d}p$ and are just as unknown. In effect, the unknown $p$ appears on both sides of Eq. (12), making it an integral equation to be solved rather than just an integral of a known function.

**Expansion of Sources**

Lighthill sidestepped the problem by replacing the local $\rho$ by the ambient $\rho_0$ in $q$ [Eq. (11)] (and, in effect, the local $p$ by $p_0$ to eliminate the second term). We can explore the significance of this approximation by carefully expanding the source terms $q$ with use of the conservation Eqs. (9) and (10) and the homentropic gas law Eq. (14). In this expansion, we assume that $v_i$ is compounded of a transversely sheared mean flow $U(x_2)$ or $U(y)$ with superimposed turbulence $u_i(\boldsymbol{x})$

$$v_i = U_i + u_i; \quad U_i = (U(x_2),0,0) \quad i = 1,2,3$$
$$x_1, x_2 = x, y \text{ also} \tag{15}$$

Our expansion (one of many possibilities) converts Eq. (11) into

$$\left[\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]p = \underbrace{2\rho\frac{\mathrm{d}U}{\mathrm{d}y}\frac{\partial v}{\partial x}}_{\text{shear}} + \underbrace{\rho\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j}}_{\text{self}} \underbrace{- \frac{1}{c^2}\frac{\mathrm{D}^2 p}{\mathrm{D}t^2} + \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}}_{\text{wave convection}} \tag{16}$$

plus other terms that are unimportant for an unheated jet; $v$ has been written for $u_2$ and $y$ for $x_2$ to simplify the notation. (In a round jet $v$ is the radial component of $u_i$ and $\mathrm{d}U/\mathrm{d}y$ is replaced by $\mathrm{d}U/\mathrm{d}r$.)

The self-noise term in Eq. (16) is essentially the same as that in Eq. (6), by virtue of Eq. (4). The shear-noise term, although different from that in Eq. (6), can be shown to be equivalent in an integral sense. The *third* source term in Eq. (16) accounts for convection of the sound waves (but not the sources) by the flow. This can be seen by transferring the composite term to the left-hand side. The net effect is replacement of

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} \quad \text{by} \quad \frac{1}{c^2}\frac{\mathrm{D}^2 p}{\mathrm{D}t^2}$$

incorporating the convective derivative $\mathrm{D}/\mathrm{D}t$ and the local sound speed $c$.

**Interpretation of Sources**

A schematic interpretation of this expansion of the Lighthill source terms is given in Fig. 5. These are not (except for the self-noise term) "real" sources that generate sound within the jet; such sound in passing through the jet would suffer refraction (by mean flow velocity/temperature gradients) and scattering (by turbulence velocity/temperature gradients). Instead they are "virtual" sources—including the jet flow itself as a source component—that replace the jet as a source of sound.

The virtual sources are correlated with the self-noise real sources; they simulate, among other things, the effects of refraction and scattering so as to yield the correct sound

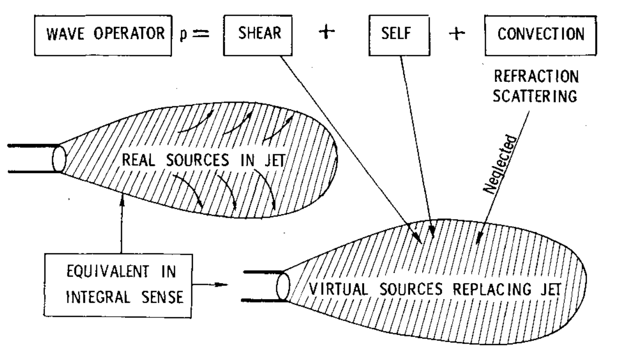



**Fig. 5 Interpretation of expanded Lighthill sources, Eq. (16); the virtual sources include the jet flow itself as a source component.**

pattern emerging from the jet. However, Lighthill and his followers, in effect, suppress the wave convection terms in Eq. (16) that account for refraction and scattering. The replacement of $\rho$ by $\rho_0$, etc., alluded to earlier, is equivalent to this neglect. On the theoretical side, this is a significant defect in the Lighthill approach. On the practical side, it leads to significant error (overprediction) in the noise only near the jet axis, an unimportant region of the patterns.

The "sources" on the right-hand side of Eq. (16) are not unique. In fact, dozens of equivalent (and nonequivalent) source term expansions have been published by flow noise researchers. *This multiplicity of competing source terms has been a major contributor to confusion for at least the last decade.* The sources of Eq. (16) permit relatively simple physical interpretation, hence their choice in the present section. However, alternative choices have other virtues, as will be seen later.

### Shear: Propagation Role vs Source Role

Let us return to Eq. (16) and shift source terms from the right-hand side to the left-hand side

$$\underbrace{\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p}_{\text{Lighthill equation}} = \boxed{\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{1}{c^2}\frac{D^2 p}{Dt^2}} + 2\rho\frac{dU}{dy}\frac{\partial v}{\partial x} + \rho\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} + (\text{other}) \qquad (16)$$

$$\underbrace{\frac{1}{c^2}\frac{D^2 p}{Dt^2} - \nabla^2 p}_{\text{convection}} \underbrace{- 2\rho\frac{dU}{dy}\frac{\partial v}{\partial x}}_{\text{shear}} = \underbrace{\rho\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j}}_{\text{self}} + (\ ) \qquad (17)$$

"Lilley" equation

This changes a form of the Lighthill equation into a precursor of Lilley's[18] wave equation.* The chief difference is that both wave convection (accounted for in $D^2p/Dt^2$), and mean-flow shear now appear on the left-hand side as *wave propagation* operators.[19]

The Lilley equation has been used widely in recent years in attempts at jet noise prediction. Its proponents argue—with justification—that it more nearly describes the physics of the noise generation process: the right-hand side is identified as the real source pattern, and the left-hand side is interpreted as locally enforcing the proper wave propagation of the sound from these sources. This is in contrast to the virtual sources of the Lighthill approach (Fig. 5).

The reader may recall that the shear term, as well as the self term, of Eq. (16) is *modeled* in the Lighthill approach (statistically, in terms of two-point correlations); but the "Lilley" Eq. (17) seems to be telling us that when we *model* the self term, Eq. (17) will, in effect, *calculate* the shear term for us. There is, however, no real inconsistency in the two approaches: they differ only in convenience and in details that affect accuracy.

In this context, I must restate the obvious as a principle of fundamental importance, often overlooked: *the wave equation*—either Lighthill or "Lilley"—*in concert with the momentum equations governs the entire turbulent flow*. The wave equation supplies the information of the continuity equation: reversal of its derivation via the momentum equations yields that equation up to a constant of integration. *The notion that the right-hand side is "turbulence" driving the left-hand side as "sound" is oversimplified*: it is a valid interpretation only outside the flow. Within the jet all terms of the wave equation refer to the actual turbulent flow. Thus, the velocity component $v$ in the shear term of Eq. (11) is not just that of an *acoustic* wave, but includes also that of the local *turbulence*; the former would be so much weaker than the latter at jet noise levels as to be virtually immeasurable.

It follows that the wave and momentum equations serve to *correlate* the self- and shear-noise terms. Either the *equations* or *flow measurements* can provide the shear/self correlation. Indeed, flow measurements exploit the turbulent flow itself as an analog computer to determine compatible values. If we use *flow measurements* for the shear term we are modeling it in the source role[1,2,11,12]; if we use *equations* (i.e., shear term on left-hand side) it plays a propagation role (Lilley approach[18]). The choice largely reduces to a question of relative convenience and accuracy, and tradeoffs between the two.

On the question of accuracy, the propagation role (Lilley equation) would seem in some respects to have an edge over the source role. In the latter role the pressure or density correlations needed for full evaluation of all the terms are at the limits of current measurement techniques. Thus, it is most common to neglect the wave-convection source terms, with consequent failure to predict the *refraction* dimple. Also, the compatibility between the self- and shear-noise correlations is normally based more on oversimplified assumption than on measurement.

The greater apparent potential for predictive accuracy with the Lilley equation, however, has not been demonstrated credibly. This is shown, for example, in the numerical comparisons to follow. The loss of accuracy probably arises, in part, because of simplifying approximations forced by enormous difficulties that impede a rigorous solution.

#### *Propagation Role (Mani) vs Source Role (Ribner)*

Figure 6 compares schematically the predictive approaches of Mani[20] (shear term in propagation role) and, e.g., Rib-

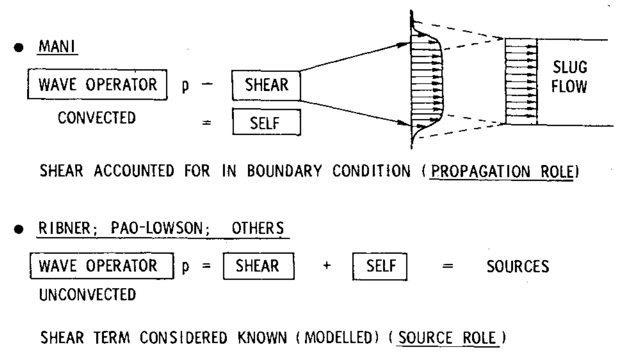



**Fig. 6 Jet noise predictive equations: propagation role vs source role for mean-flow shear.**

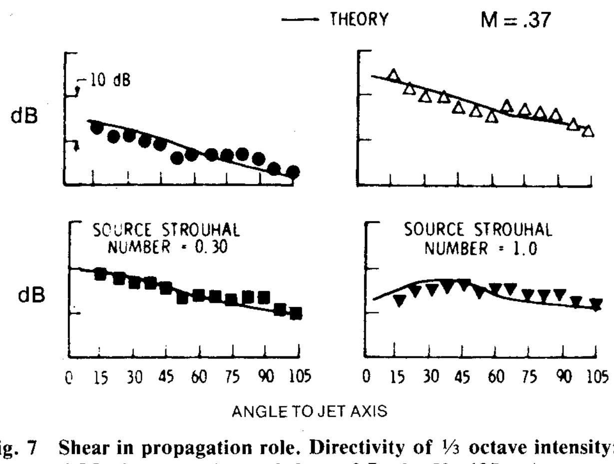



**Fig. 7 Shear in propagation role. Directivity of ⅓ octave intensity: theory of Mani vs experimental data of Lush; $U = 125$ m/s, source Strouhal number a) 0.03; b) 0.10; c) 0.30; d) 1.0; ——theory.**

*Lilley applies a further $D/Dt$ operation throughout, so as to replace $v$ by $p$ by use of the momentum equation.

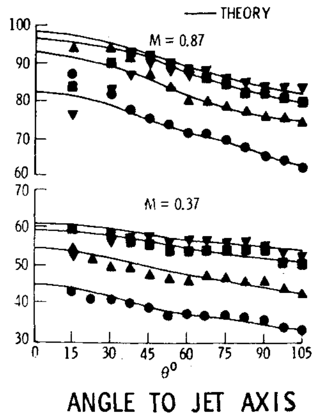



**Fig. 8 Shear in source role. Directivity of ⅓ octave intensity: theory of Ribner vs experimental data of Lush; a) $U=300$ m/s, b) $U=125$ m/s; source Strouhal numbers 0.03 ●; 0.1 ▲; 0.3 ■; 1.0 ▼; — theory.**

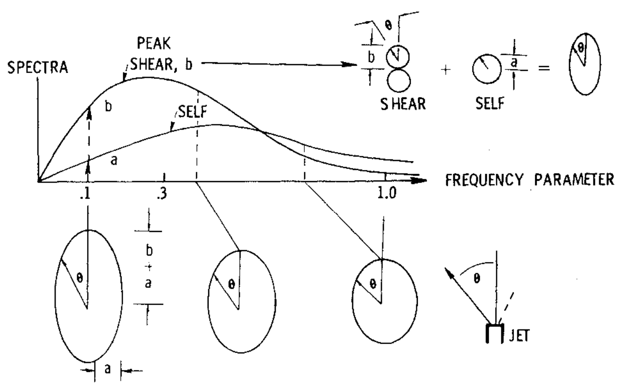



**Fig. 9 Shear in source role. Frequency-dependent interplay between shear and self-noise directional patterns decreases axial beaming with increased frequency.**

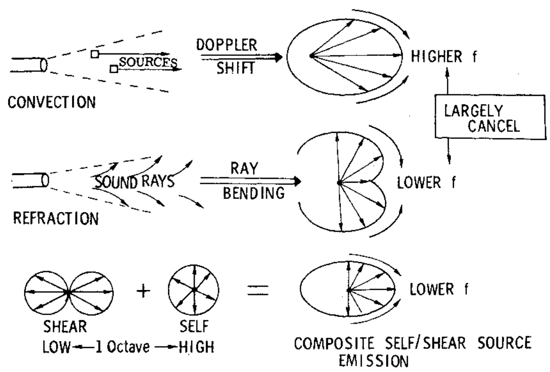



**Fig. 10 Shear in source role. Construction to explain reverse Doppler shift paradox: the relatively low pitch of the downwind jet noise.**

ner,[12] Pao and Lowson,[21] and Moon and Zelazny[22] (shear term in source role). The shear term does not appear per se in Mani's model, in which the shear layer is squeezed into a velocity discontinuity: the shear term (which, strictly speaking, is now infinite) is replaced by an equivalent interface boundary condition, "continuity of particle displacement."[23,24] The equivalence is proved in Ref. 19.

*Mani vs Ribner: Directivity*

Figures 7 and 8 compare quantitative predictions of jet noise overall directional patterns by Mani's and Ribner's methods with experimental measurements of Lush.[27] In the case of Mani (Fig. 7) the vertical height of the pattern (absolute level) is adjusted for best fit; in the case of Nossier and Ribner[25,26] (Fig. 8), the absolute levels are tied together at 90 deg to the jet axis ($\theta=90$ deg); there is not much difference. These two predictive theories, taking very different mathematical forms, and with very different roles for the shear term, are seen to agree quite well with each other and with the experimental measurements.

Both roles for the shear term (*propagation*: Mani; *source*: Ribner; Pao-Lowson; Moon-Zelazny) lead to enhanced "beaming" of the sound downstream at low frequencies (Fig. 9). Mani, and Goldstein,[28] attribute this to an increase in the exponent of the convection factor $C^{-5}$ at low frequencies (to $C^{-9}$). Ribner, on the other hand, associates the effect with a frequency-dependent interplay between the shear- and self-noise directional patterns. There is good agreement in the downstream directions, but an apparent discrepancy in the upstream direction.

### Shear in Source Role: Further Properties

*Reverse Doppler Shift Paradox*

The sound sources in a jet—the turbulent eddies—are convected downstream by the flow. Hence one would expect to hear *higher* frequencies downstream than to the sides ($\theta=90$ deg), owing to Doppler shift. Actually, the reverse is true: the jet noise exhibits *lower* frequencies downstream. I have called this the "reverse Doppler shift" paradox.

Figure 10 provides an explanation.[29,30] Doppler shift indeed increases frequencies downstream *when acting alone*; but refraction by the jet velocity and (when heated) temperature gradients turns the sound rays away from the axis, more strongly with increasing frequency. The convection (Doppler) and refraction effects on dominant frequency vs direction tend to cancel. Finally, the combination of shear- and self-noise sources is a mix of two frequency patterns, low and high, associated with very different directivities. The figure shows how the low frequencies dominate in the downstream direction.

*Match of Self and Shear Spectra*

Figure 10 repeats that the self-noise spectrum should lie about an octave above the shear-noise spectrum, and analytical theory further predicts similar spectral shapes. These two spectra can be extracted[25,26,31] from experimental jet-noise measurements at two distinct angles from the jet axis by means of a narrow band analog[12] of Eq. (7). The choices $\theta=45$ deg, $\theta=90$ deg are suitable, because they avoid the refraction-dominated zone ($|\theta| \leq 35$ deg, say) not accounted for in the theory.

The spectra in Fig. 11 were obtained in this manner from data of Refs. 32-34 normalized to unity peak value, and the self-noise curve down-shifted one octave. The collapse of the curves strikingly confirms the *similar shape/one octave shift* prediction. Further confirmation has come from recent work of Richarz.[35] He extracted self- and shear-noise spectra from cross-correlations of laser-Doppler velocity signals in the jet with far-field microphone signals; again the spectra showed a very good collapse after the downshift.

We have seen that the propagation role for the shear term in the hands of Mani[20] leads to numerical results in fairly quantitative agreement with those for the source role (Fig. 7 vs Fig. 8). Thus a similar extraction procedure for the self- and shear-noise spectra, as we have defined them,[12,26] should likewise lead to a good collapse of the spectral shapes. However, Mani's theory in itself contains absolutely no guidelines to lead one to expect such a match; the phenomenon is quite concealed in the mathematics and is extracted only by numerical evaluation. In this respect, the source role provides much more guidance.

### Shear in Propagation Role: Instability Waves

Mean flow shear long has been known to mediate instability in quasiparallel flows. This occurs whether the shear is

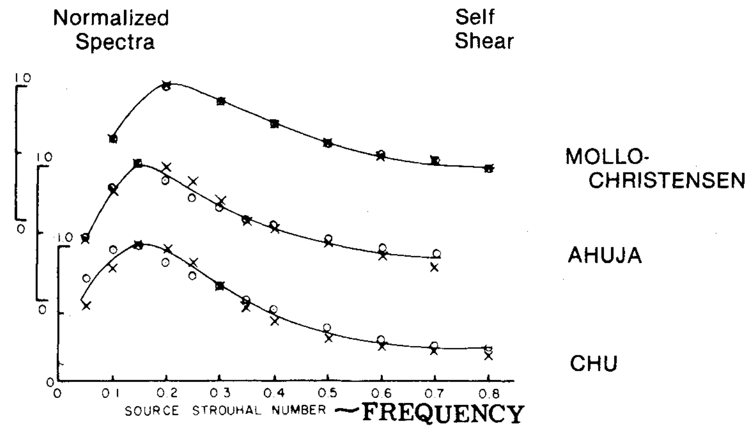



**Fig. 11 Shear in source role. Match of experimentally derived normalized shelf/shear basic noise spectra.**

distributed in a continuous velocity profile or is infinitely concentrated as an interface between parallel flows. The homogeneous Lilley equation (right-hand "forcing" term set equal to zero) exhibits this instability, whereas the homogeneous Lighthill equation, lacking the shear term, does not.

The growth of instability waves in the bounding shear layer of a round laminar jet is a precursor to the transition to turbulence. Downstream of the nozzle the waves roll up into large vortex structures, many of them ring-like; they grow and distort as they are convected by the flow. Nonlinear effects take over, and some vortices tend to merge as they grow ("pairing"). At high Reynolds numbers (in the hundreds of thousands) the spatial-temporal irregularity that we call turbulence tends to obliterate these "large scale coherent (vortex) structures" with which it coexists. Careful conditional sampling techniques are required to render the structures visible in schlieren photographs, and to show their growth and merging.[36]

The Lilley equation, because its homogeneous form exhibits instability, has been thought by many to be much closer to accounting for flow dynamics than the stable Lighthill equation. We have, however, argued as to their full equivalence earlier, *when taken together with the momentum equations*. The essential point in calculating jet noise is that major features of the turbulent flow must be known a priori. If the turbulence is assumed known (statistically in terms of space-time correlations), then there is no need to invoke the instability waves that led to its generation.

In the case of the Lilley equation, the appearance of instability waves has nothing to do with jet noise: it is merely an artifact of the Green's function method of solution. The Green's function is the solution obtained when the right-hand-side source distribution is replaced by a localized impulse ($\delta$ function). The physical relevance is very remote, even though the formal solution for the pressure field is a convolution of the Green's function with the source distribution. If this solution refers to the pressure field *within* the jet, it must be that of the turbulent flow, and in that flow (at high Reynolds numbers) instability waves, as mentioned, are hardly discernable. Thus there must be a high degree of *cancellation* of instability waves in the Green's function convolution integral.

The considerations of the last paragraph put the Lighthill equation in a much more favorable light. The lack of instability in its Green's function is an asset in jet noise prediction, not a liability. It obviates some very formidable difficulties. Another advantage of great appeal is the utter simplicity of the Green's function. It is the well-known point monopole solution of the ordinary acoustic wave equation. For simple models of the turbulent source statistics this permits simple *closed-form* solutions for radiated pressure field [e.g., Refs. 12 and 21]. With the Lilley equation this is not even remotely possible.

The discovery of large scale coherent structures traceable to instability waves has stimulated great interest among jet noise researchers (e.g., Refs. 36-39). The early expectation was that the large scale structures might dominate over the general turbulence in generating the jet noise. However, a common view now is that "they do not directly radiate significant acoustic power in a subsonic jet, but do govern the production of the turbulent fluctuations which radiate broadband jet noise."[36]

### *Pure Tone Forcing and Jet Noise Amplification*

A new and very dramatic manifestation of instability waves was only recently discovered by Bechert and Pfizenmaier[40] and by Moore.[36] They injected strong pure tones into the plenum above a jet nozzle by means of loudspeaker drivers. When the driving pressure exceeded a certain threshold level (0.08% of the jet dynamic pressure[36]) the broadband jet noise spectrum was found to be increased; this was in addition to the pure tone "spikes" (fundamental plus harmonics). This behavior was found to be quite frequency dependent: the frequency must be within a restricted range. The jet noise amplification was found to be associated with an increase in turbulence levels.

Various internal sources of engine noise—the turbine, fan, wakes, combustion—could, in light of the preceding, amplify the jet noise in addition to their direct contribution. This has been put forward as a possible explanation of the variability of jet noise measurements between different rigs.[36] This difficulty in duplicating measurement levels, even under laboratory conditions, has been a puzzle.

Shear layer stability calculations by Michalke[41-43] and by Chan[44] seem to be relevant to the behavior for below-threshold excitation.[36] A more recent theory by Tam and Morris[45] modifies the stability theory for parallel flow to deal with slightly divergent flows; more importantly, it goes beyond the stability theory to predict the acoustic radiation. Numerical results agree qualitatively with observed noise directional patterns for supersonic jets.

### *Impulsive Forcing and Pseudosound Amplification*

Impulsive excitation of a jet likewise stimulates instability waves. (In the linear regime, of course, preceding the nonlinear rolling up process, impulsive and pure tone excitation can be related by Fourier transform.) Bayliss and Maestrello[46,47] have employed impulsive excitation in a very graphic numerical study. In a departure from the Lighthill or Lilley approaches, they did not set up a wave equation. Instead, they linearized the inviscid conservation equations about a specified mean velocity profile. This was taken as the measured profile of a jet and allowed for the jet spreading; this was an advance over the "locally parallel" assumption in earlier jet-noise work. An impulsive point source of mass near the nozzle was specified as providing the jet excitation.

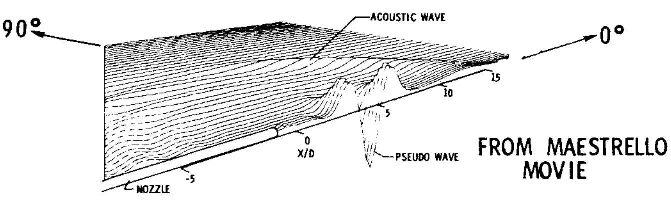



**Fig. 12 Impulsively excited mathematical jet: calculated differential pressure field attributable to shear terms (after Bayliss and Maestrello[47]). Acoustic wave null at 90 deg agrees with figure 8 pattern (Figs. 3 and 10) predicted for shear noise radiated by turbulence (source role).**

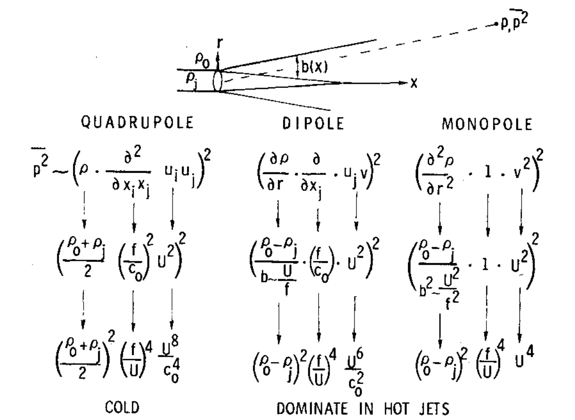



**Fig. 13 Scaling of source term contributions to mean square sound pressure for hot jets in terms of density differences (after Mani[49]).**

Some of their computer printouts[47] showed successive instantaneous calculated pressure patterns in a plane through the axis. Several sequences of these patterns were photographed as a two-minute computer movie. By courtesy of Dr. Maestrello this was presented at the oral version of the present lecture. The movie exhibits three sequences: each shows a central nonpropagating disturbance (pseudosound), with a growing circular wave propagating therefrom. The appearance is much like the central disturbance and growing ripple that follows from dropping a pebble in still water; however, in the present case both central disturbance and ripple are swept downstream by the jet flow.

The first sequence utilizes the complete equations; it shows the central pseudosound disturbance growing to high amplitude, and then decaying as it travels. The second sequence shows the pattern with the mean-flow *shear terms*—those equivalent to the shear term of the Lighthill equation —*omitted* in performing the computer calculation. Although the sound wave (expanding circular ripple) looks rather similar to that with the shear terms included, the pseudosound wave shows a great difference; it decays shortly after its impulsive creation, showing no instability-induced amplification.

*Evidence of Shear Noise Figure 8 Pattern*

The pressures in the third sequence of the computer movie are the algebraic difference of those of the first two: the plots represent the *differential pressure field attributable to the presence of the shear terms.* A typical frame from this sequence is shown in Fig. 12. The amplitude of the acoustic wave (circular ripple) is seen to vanish at $\theta = 90$ deg; it appears to conform qualitatively to the figure-eight, dipole-like, intensity pattern predicted with shear as a source term (Figs. 3 and 10). This is perhaps the most graphic evidence of such a dipole-like behavior to date. Note also the consistent apparent reversal of phase in the upstream direction. [The slight loss in amplitude in the range $|\theta| \leq 20$ deg must be associated with refraction (cf. Figs. 2 and 10), not allowed for in the source role modeling.]

The Bayliss-Maestrello approach,[46] with the shear term in a propagation role, takes a very different form from the Lighthill approach, with the shear term in a source role. Moreover, they are applied to different phenomena; the former to a pulsed, turbulence-free mathematical jet, the latter to the noise from jet turbulence. Yet the respective predictions in the two cases seem to be in close agreement on the shear-noise directivity.

**Hot Jets: Mani Approach**

Morfey[48] showed how the jet radial density gradient $d\rho/dr$ in a heated jet could yield additional jet noise. This gradient serves as a strength factor for new Lighthill equation source terms. Mani[49] approached the hot-jet problem more quantitatively from the viewpoint of the Lilley equation. Actually, he used a wave equation equivalent to

$$\frac{1}{c^2}\frac{D^2 p}{Dt^2} - \nabla^2 p - 2\rho \underset{①}{\frac{dU}{dy}\frac{\partial v}{\partial x}} - \underset{②}{\frac{1}{\rho}\frac{d\rho}{dy}\frac{\partial p}{\partial y}} = \rho\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} \equiv \text{sources} \tag{18}$$

and in effect expanded the right-hand side with $\rho = \rho(y)$ for a plane jet ($y \to r$ for a round jet), into

$$\text{sources} = \frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j} - 2\underset{③}{\left(\frac{d\rho}{dy}\right)\frac{\partial u_j v}{\partial x_i}} - \underset{④}{\left(\frac{d^2\rho}{dy^2}\right)v^2} \tag{19}$$

where $v$ is the velocity component in the $y$ direction, and perturbations of $\rho$ from the local time average are ignored.

This exhibits a new *dipole*-source term $\sim d\rho/dy$ and a new *monopole*-source term $\sim d^2\rho/dy^2$. The first term of Eq. (19) is identical with Lighthill's, implying *quadrupoles* of strength density $\rho u_i u_j$. (The space derivatives $\partial/\partial x_i$, $\partial^2/\partial x_i \partial x_j$ dictate the dipole and quadrupole characters, respectively; the explanation may be found in, e.g., Refs. 1 and 11.)

Equations (18) and (19) were derived for smooth mean-velocity and density profiles, $U = U(y)$, $\rho = \rho(y)$, but for his further analysis Mani goes over to a "slug flow" model of a jet. Within the jet $U$ and $\rho$ are uniform at values $U_j$, $\rho_j$, and there is a discontinuous jump to respective values 0, $\rho_0$ in crossing the jet boundary. At the discontinuity the gradient terms ①, ②, ③, and ④ would all be infinite, and they would vanish everywhere else. However, Mani points out that ① may be replaced by an equivalent boundary condition "continuity of particle displacement."[23,24] The equivalence, as mentioned earlier, is proved in Ref. 19, and by similar methods one can show that *this same boundary condition* replaces term ② as well.

The dilemma of terms ③ and ④ is dealt with by estimating $d\rho/dy$, $d^2\rho/dy^2$ for a realistic smooth-profile jet flow, not the square-profile slug flow. The terms are not really evaluated, but rather scaled with the exterior-interior density difference $(\rho_0 - \rho_j)$ divided by the local width of the mixing region. It is hard to assess the validity of this apparently inconsistent dual-flow model. It strikes me as extremely clever, but I remain uneasy about the inconsistent treatment of the other density-gradient term ②.

Mani brings out the relative importance of the source terms with factors $d\rho/dy$, and $d^2\rho/dy^2$ in terms of scaling laws. A basic step involves the equivalence, in an integral sense, of source-term space and time derivatives. Thus for computing sound pressure in the far field, $\partial/\partial x_i$ may be replaced by $\partial/c_0\partial t$ times a directional factor (Appendix of Ref. 50). For scaling purposes, the last factor may be dropped and $\partial/\partial t$ taken proportional to a typical frequency $f$ in the jet, say the local spectral peak frequency. Thus the final scaling is

$$\partial/\partial x_i \sim f/c_0 \tag{20}$$

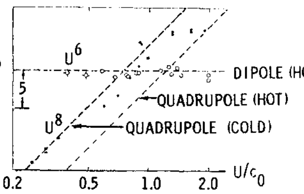



**Fig. 14 (Velocity)$^6$ scaling of hot jets (after Tester and Morfey[51]; compatible with Mani[49]).**

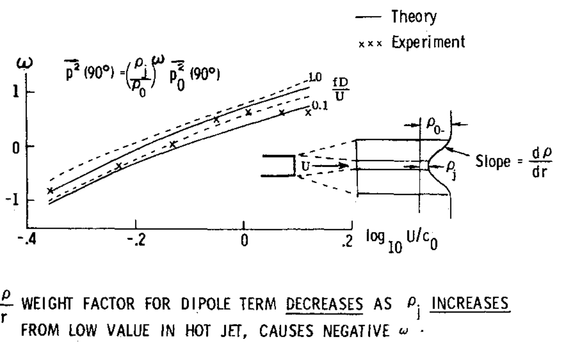



**Fig. 15 (Density)$^\omega$ scaling of hot jets (after Mani[49]). Weight factor $d\rho/dr$ of dipole term decreases (Fig. 13) as $\rho$ increases from low value, causes negative $\omega$.**

*(Velocity)$^6$ Scaling of Hot Jets*

With the aid of Eq. (20) the scaling is developed schematically in Fig. 13 for a round jet $(y \to r)$. The turbulent velocities $u_i$, $u_j$ scale with nozzle velocity $U$, density $\rho$ with average density $(\rho_0 + \rho_j)/2$, and density gradient $d\rho/dr$ with $(\rho_0 - \rho_j)/b$. The local width of the mixing region $b$ scales with (local velocity/local frequency). This, in turn, scales with (nozzle velocity/peak frequency) $= U/f$ in the scaling laws. The scaling relations of Fig. 13 then follow. A factor $L^3D^3$ ($L$ = average "correlation length" of sources), resulting from the integration over the jet, remains to be introduced [cf. Eq. (5)], giving the final source term contributions as

$$\overline{p^2} \sim \underbrace{\left(\frac{\rho_0+\rho_j}{2}\right)^2 \left(\frac{fD}{U}\right)^4 \frac{U^8}{c_0^4} D^2}_{\text{quadrupole}}$$

$$\underbrace{(\rho_0-\rho_j)^2 \left(\frac{fD}{U}\right)^4 \frac{U^6}{c_0^2} D^2}_{\text{dipole}}, \quad \underbrace{(\rho_0-\rho_j)^2 \left(\frac{fD}{U}\right)^4 U^4 D^2}_{\text{monopole}} \tag{21}$$

The nondimensional factor $fD/U$, the peak frequency Strouhal number, essentially is invariant. Thus the equation exhibits the well-known sequence of velocity scalings: $U^8$, $U^6$, $U^4$ for quadrupole, dipole, and monopole flow noise generators, respectively. The dipole and monopole strength densities are nonzero only when there are density gradients, e.g., for heated jets. We form the ratio

$$\frac{\text{quadrupole emission}}{\text{dipole emission}} \sim \left(\frac{1+\rho_j/\rho_0}{1-\rho_j/\rho_0}\right)^2 \frac{1}{4} \frac{U^2}{c_0^2} \tag{22}$$

neglecting possible differences in numerical factors. For hot low-speed jets this can be much less than unity: the dipole emission with its $U^6$ law will dominate (Fig. 14).[51]

[We discount a corresponding very small ratio quadrupole/monopole on the ground that the monopole scaling in Eq. (21) is misleading: the monopole source scaling term should be multiplied by a very small numerical factor, since $d^2\rho/dr^2$ is approximately zero in the middle of the mixing region (radially), where the turbulence intensity reaches a sharp peak.]

*(Density)$^\omega$ Scaling of Hot Jets*

The scaling laws, Eq. (21), lacking direction-dependent proportionality factors, fall far short of quantitative prediction laws for jet noise. In order to obtain such laws Mani solves, in effect, Eqs. (18) and (19), subject to the interface boundary conditions. He uses a Green's function/Fourier transform technique involving harmonic point sources moving axially with the flow. The results are complex expressions in terms of Bessel functions that ultimately are evaluated numerically.

The results of Mani's calculations are put in a form amenable for comparison with experiment. The experimentalists scale density effects by

$$\frac{\overline{p^2}(\rho_j)}{\overline{p^2}(\rho_0)} = \left(\frac{\rho_j}{\rho_0}\right)^{\omega} \tag{23}$$

for a given measurement distance, angle, and jet Mach number; thus, the exponent is a function of these parameters (and of Strouhal number $fD/U$, if the sound is filtered at center frequency, $f$). Experimental values for a hot jet are compared with predictions by the theory for a number of cases in Mani's paper.[49] A typical case, $\theta = 90$ deg, is reproduced here (Fig. 15), showing impressive agreement.

**Hot Jets: Morfey, Michalke-Michel Approaches**

For Morfey's approach[48] we must return to the basic Lighthill Eq. (11) and add labels:

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \underbrace{\frac{\partial^2 \rho v_i v_j}{\partial x_i \partial x_j}}_{Re\text{ stress}} - \underbrace{\frac{\partial^2}{\partial t^2}\left(\rho - \frac{p}{c_0^2}\right)}_{\text{excess density}} \tag{24}$$

The "excess density" measures the deviation of $d\rho$ from the uniform jet isentropic value $dp/c_0^2$, ignoring local perturbations in the speed of sound. For application to heated jets, (and gas mixtures of different densities, but similar values of $\gamma$), Morfey carried out an ingenious expansion of the term. It was found that part of this expansion cancels the Reynolds stress term and replaces it with a similar term with $\rho_0$ in place of $\rho$! Another part leads to source terms proportional to mean-flow density gradient $d\rho/dr$, as in Mani's approach.

Morfey[48] limited his applications to the development of scaling laws for the noise radiation from hot jets. It remained for Michalke and Michel[52] to develop the approach into a quantitative predictive method. They generalized and slightly corrected Morfey's analysis. Further, with the objective of allowing for *jets in flight*, they included an external flow outside the jet. In this section, however, we set this effective reversed flight velocity equal to zero.

Michalke and Michel's expansion of the source terms of the Lighthill Eq. (24) (right-hand side) takes the form

$$q = \underbrace{\rho_0 \frac{\partial^2}{\partial x_i \partial x_j}\left[\left(1+\frac{p'}{\rho_0 c_0^2}\right) v_i v_j\right]}_{\text{quadrupole}} - \underbrace{\frac{\partial}{\partial x_i}\left[p' \frac{\partial}{\partial x_i}\left(\frac{\rho_0}{\rho}\right)\right]}_{\text{dipole}}$$

$$\underbrace{-\frac{\partial^2}{\partial x_i^2}\left[\left(1-\frac{\rho_0}{\rho}\right)p'\right]}_{\text{quadrupole}} + \text{higher order} \tag{25}$$

where $p' = p - p_0$. The terms accounting for density effects (factors $\rho$), unlike the dipole term of Mani [Eq. (19)], are of *exact* dipole and quadrupole form, respectively (the operators $\partial/\partial x_i$, $\partial^2/\partial x_i \partial x_j$, respectively, precede the entire term). This

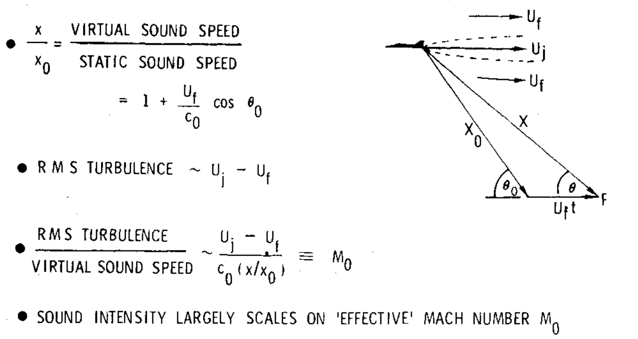



**Fig. 16 Parameters for scaling flight effects on jet noise (after Michalke and Michel,[52] with liberties of interpretation).**

in turn leads to a *formally exact* integral expression for the radiated sound pressure. (An expansion of the last term, by the way, includes the term $-(\rho_0/\rho^2)\ (\partial\rho/\partial x_i)\ (\partial p/\partial x_i)$; this differs by a factor of $\rho_0/\rho$ from the term (2) discussed in connection with Mani's approach but, taken together with other source terms here, is equivalent.)

The Lighthill Eq. (24) with the Michalke-Michel source expansion Eq. (25) is fully equivalent to the "Lilley" Eq. (18) used by Mani. Thus, it would be expected that the predicted effects of density gradients on sound generation should be similar. Like Mani, they inferred a dominance of the dipole radiation for hot jets, with a consequent $U^6$ law.

#### Flight Effects: Michalke-Michel Approach

The primary motivation of the Michalke-Michel approach to jet-noise prediction is allowance for an external stream $U_f$ to simulate flight effects. The Lighthill wave operator [left-hand side of Eq. (24)] is modified to exhibit wave convection at speed $U_f$; and in the expansion, Eq. (25), of the right-hand side $U_f$ is added vectorially to $v_i$.

Their solution for the sound field in terms of the sources Eq. (25) is hard to display very simply in absolute terms. However, the scaling from static jet to jet in flight at Mach number $U_f/c_0$ is much simpler. We can rationalize key parameters in this scaling with the aid of Fig. 16. The sound that would propagate along $x_0$ in quiescent air (static case) will be swept downstream as if it had propagated along $x$. The effective speed along the greater distance $x$ is greater than the propagation speed of sound. This virtual sound speed is defined by

$$\frac{x}{x_0} = \frac{\text{virtual sound speed}}{\text{real sound speed}} = 1 + \frac{U_f}{c_0}\cos\theta_0 \tag{26}$$

It is well verified[53] that the rms turbulence velocities scale closely with the jet-outer stream velocity difference $U_j - U_f$. Thus we may form the ratio

$$\frac{\text{rms turbulence}}{\text{virtual sound speed}} \sim \frac{U_j - U_f}{c_0(x/x_0)} \equiv M_0 \tag{27}$$

Now the Lighthill integral for sound pressure radiated by a static jet scales with the turbulence/sound speed ratio, and this in turn scales with the ratio $U_j/c_0 = M_j$. Thus, for flight, $M_0$ seems to behave like an *effective* Mach number, largely replacing $M_j$ for noise scaling.

The replacement is not complete, however, because the jet flowfield is stretched downstream[52,54] in flight by a factor $\alpha$. On the basis of a physical argument, $\alpha$ is modeled as

$$\alpha = 1 + \frac{AU_f}{U_j - U_f} \tag{28}$$

$$1.5 \le A \le 3 \tag{29}$$

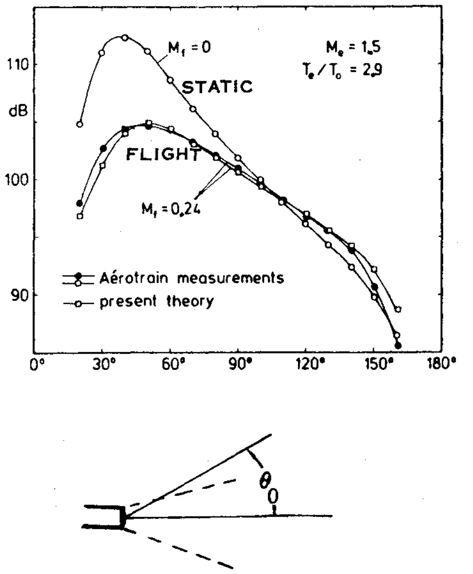



**Fig. 17 Scaling of static jet noise directional pattern to pattern in flight (after Michalke and Michel[52]; experimental data from Drevet et al.[56]).**

(More recent work adjusts the value of $A$ to 1.4). Both the jet volume and source correlation volume $L^3$ are increased by the axial stretch factor $\alpha$; their product governs the sound radiation intensity [Eq. (13)], yielding a factor $\alpha^2$. A further amplification by the (Doppler factor)$^2$, which equals $(x/x_0)^2$, is required.[55]

The final scaling law for the intensity $I_{\text{flt}}$ at point P, Fig. 16, for a jet in flight is obtained as

$$I_{\text{flt}}(x,\theta,M_j,M_f) = \left(\frac{x}{x_0}\right)^2 \alpha^2 I_{\text{ref}}(x_0,\theta_0,M_0); \quad M_0 = \frac{M_j - M_f}{x/x_0} \tag{30}$$

where $I_{\text{ref}}$ is identified as the intensity at angle $\theta_0$ and distance $x_0$ of the same jet engine operated statically (same temperature), with jet velocity $M_0/c_0$. Thus $I_{\text{flt}}$ is scaled from the values of $I_{\text{ref}}$ corresponding to a *family* of static jets. The reference static Mach number changes with $\theta_0$ through the dependence of $x/x_0$ on $\theta_0$ [Fig. 16 and Eq. (26)].

Data for a *family* of reference static jet directivities can be generated from measurements at a single value of $M_0$, if the Mach number scaling law is known. For a hot jet, Michalke and Michel employed $I_{\text{ref}} \sim M_0^6$, inferred from the analysis discussed earlier. Insertion in the static→flight scaling law Eq. (30) has yielded very impressive agreement with flight measurement in a large number of cases; one example is shown in Fig. 17 (data from Ref. 56).

When quadrupole dominance appropriate to a cold jet was assumed, implying $I_{\text{ref}} \sim M_0^8$, the static→flight scaling was found to be much poorer.[57] Thus we have further evidence, albeit indirect, of a $U^6$ scaling law for hot jets.

## Suppression

A large number of schemes for suppressing jet noise have been tried, either ad hoc or motivated by simplistic notions. As we have seen, the theory for even a simple round jet is so complex that it has provided little guidance. An important exception is the principle of the coannular or fan jet engine, to be discussed later.

#### Multitube and Corrugated Suppressors

An early, but still viable, example of a moderately successful device is the corrugated nozzle suppressor, or the largely equivalent multiple-tube suppressor (Fig. 18). The

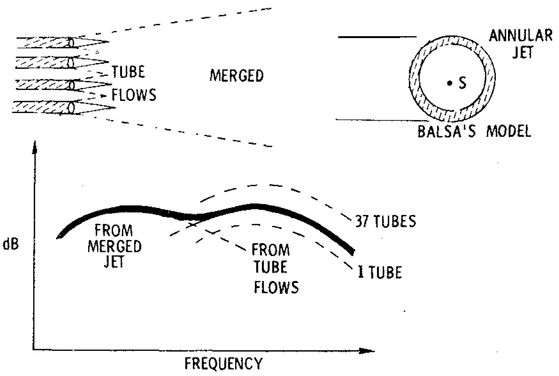



**Fig. 18 Multitube jet noise suppressor: shielding of inner jets by outer jets according to theory of Mani and Balsa.[58]**

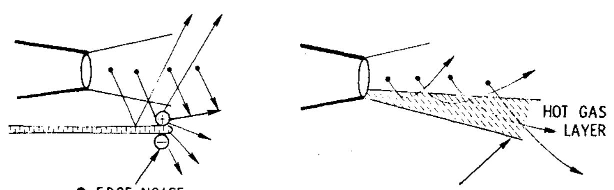



**Fig. 19 Shielding of jet noise: plate or wing vs thermal layer. Edge noise and entrainment, respectively, limit effective shield length.**

individual tube flows are distinguishable near the nozzle and dominate the high-frequency part of the spectrum; further downstream they merge into a large single jet, which dominates the low end of the spectrum.

If noninterfering, $n$ tubes should radiate $n$ times the acoustic power radiated by a single tube. Experimentally, the gain is far less than $n$-fold (Fig. 18). Mani and Balsa[58] explain this as an acoustic shielding of the inner jets by the ring of outer jets. They model this ring as an annular jet; moving along the axis is an oscillatory source that simulates a moving eddy of a single interior jet. Their analysis confirms the expected shielding effect. Referring to Fig. 18, this accounts for a substantial reduction in the high-frequency hump in the spectrum. A second hump is provided by sound emission from the slower, larger scale, merged jet, which swamps the tube-flow emission at low frequencies.

### Shielding: Plate vs Hot Gas Layer

Placing a jet engine above a wing can provide a measure of shielding of the jet noise. The wing acts as a reflective barrier for impinging sound (Fig. 19). The effectiveness is, as for all barriers, seriously limited at the lower frequencies by diffraction around the edge. Furthermore, that part of the jet downstream of the wing (or plate) edge—the emitter of low frequencies—is left unshielded. If we extend the wing edge or plate edge downstream to provide the shielding, a new phenomenon defeats us: an aeroacoustic interference effect generates additional dipole noise at the plate trailing edge (see, e.g., Refs. 59 and 60). This augments the low-frequency part of the spectrum, and with increasing extension can dominate over the jet noise: the increased low frequencies more than offset the reduced high frequencies.

A high-speed-of-sound gas layer under a jet can play much the same role as a solid plate barrier.[61-64] It may be either heated air or a foreign gas, and in either planar or half-round configuration. The high-frequency sound rays from the jet are sharply turned or refracted by the speed-of-sound gradients so that those near the nozzle emerge upward. This is equivalent to reflection. Further downstream entrainment of air by the spreading layer weakens the gradients and, hence, the degree of refraction. Thus, like the wing or plate shield, the effective

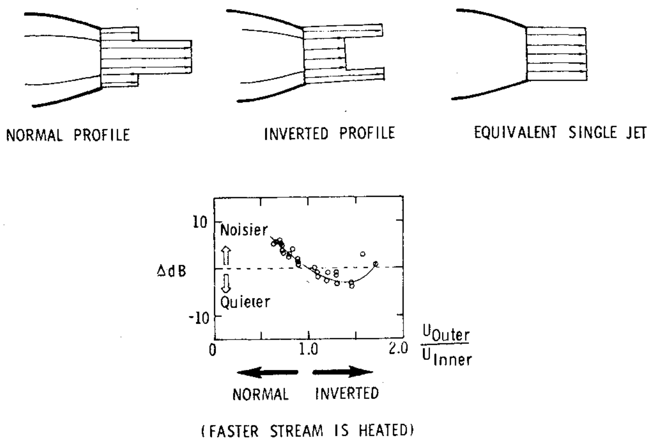



**Fig. 20 Relative peak noise intensity of coannular jets as function of $U_{outer}/U_{inner}$ at constant thrust, mass flow, and area; faster stream heated; $A_{outer}/A_{inner} = 0.85$ (after Ref. 65, data from Ref. 66).**

downstream length, and hence the realizable shielding, seems rather limited for practicable geometries but such shielding can still provide useful noise reduction at the higher frequencies.

### Coannular Jets

#### *Diameter/Velocity Tradeoff*

We have seen that quadrupole jet noise scales as $U^8D^2$, whereas thrust scales as $U^2D^2$ (omitting density factors). Thus noise intensity per unit thrust scales as $U^6$ or $D^{-6}$. The message is clear: increases in diameter $D$, which allow reductions in $U$ at constant thrust, will act powerfully to diminish jet noise. This is most effective when the increased diameter jet is uniform; however, there are still noise advantages when the larger diameter is associated with coannular jets of differing velocity. This was recognized in the early fifties and was part of the motivation for the successive generations of bypass or fan jets, each with increased diameter of the outer annulus. These have the dual advantages of greater fuel efficiency *and* reduced jet noise.

#### *Inverted Velocity Profile*

The "normal" velocity profile of the fan jet exhibits the low velocity fan flow in the outer annulus (Fig. 20), surrounding the much faster jet flow in the core. Recently it has been discovered that rerouting the flow so that the faster flow is in the outer annulus (*inverted profile* of the sketch) can achieve significant noise reductions of the broad-band peak noise level (see Fig. 20).[65,66]

For the shock-free flow conditions referred to in the figure, the reductions are principally at low frequencies and small angles to the jet axis. These are governed by emission from the "post-merged" region wherein the outer and inner flows have blended: in this region "the eddy convection velocity, source region velocity, and axial turbulence level of the coannular jet are significantly lower than those of the fully mixed jet."[65] Thus the noise reduction is seen to be a result of fluid dynamic consequences of the inverted velocity profile; these are of a kind that would not have been apparent a priori. Heating of the faster jet, by the way, plays a negligible role.[65]

The decibel reductions provided by the inverted profile for shock-free flow conditions refer to peak mean square sound pressure, not to perceived levels as the ear would hear them (PNdB). On a PNdB basis Tanna[65] found that the effect of the inverted profile varied from no change up to several PNdB *increase*. The increase results because the PNdB formula weights the higher audio frequencies more strongly than the lower, and this range of frequencies is actually enhanced by the profile inversion.

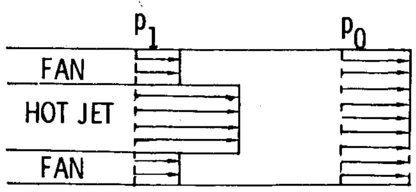



Fig. 21 Internal mixer improves coannular jet at constant diameter. Reduces noise: uniform jet quieter; increases fuel efficiency: $p_1 < p_0$. Implemented in new Pratt and Whitney JT8D-209 engine.

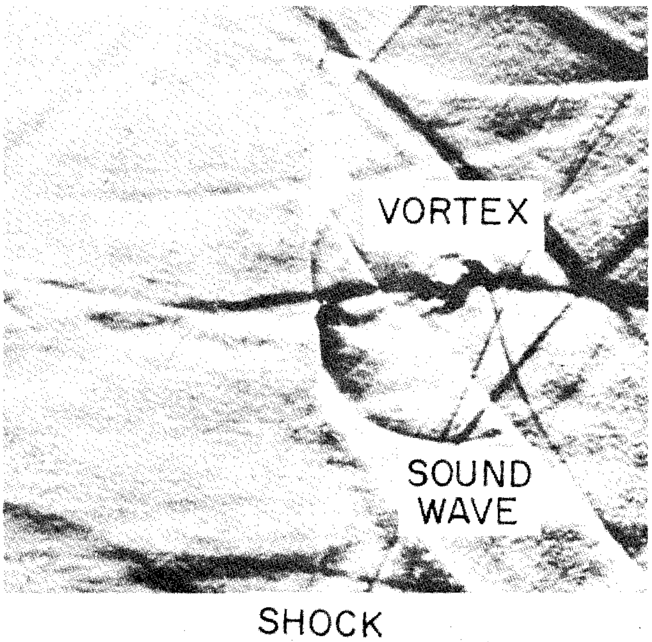



Fig. 22 Idealized jet shock-noise mechanism: passage of columnar vortex through shock generates growing cylindrical sound wave (after Hollingsworth and Richards[71]).

*Internal Mixer*

The coannular jet reduces jet noise at constant thrust via reductions in average jet velocity associated with increases in jet diameter. But, as touched on earlier, the simple scaling laws, noise power $\sim U^8D^2$, thrust $\sim U^2D^2$, imply that the noise reduction is greatest for a given diameter when the jet velocity is uniform. This is supported by experiment so long as $U_{\text{outer}}/U_{\text{inner}} \leq 1$ (Fig. 20). It fails for inverted profiles, due to changes in the aerodynamics mentioned earlier.

This principle can be used to reduce the noise of a normal profile coannular jet (cf. Fig. 20) by mixing the two jets within the nacelle to achieve a uniform efflux velocity.[67] Figure 21 illustrates the arrangement schematically, along with the benefits. The reduction of $p_I$ below the ambient $p_0$ can be deduced from a momentum balance. The reduced $p_I$ improves the engine thermodynamic work performance.

## Shock Noise

*Mechanism*

As far back as the early fifties, it was predicted that turbulence passing through a shock wave would generate intense noise.[68-70] This sound generation may be traced to the turbulence-perturbed shock pressure-jump conditions; however, the mathematical analysis is extremely complex. Since an imperfectly expanded supersonic jet contains shock waves in addition to the turbulence, this "shock noise" mechanism comes into play. It can often override the ordinary jet mixing noise.

An idealized deterministic example of this mechanism is the passage of a columnar vortex broadside through a shock. This generates a growing cylindrical sound wave downstream of the shock (Fig. 22, after Ref. 71). Theory[72] and experiment[73] show good quantitative agreement on the variation of pressure around the cylindrical wave.

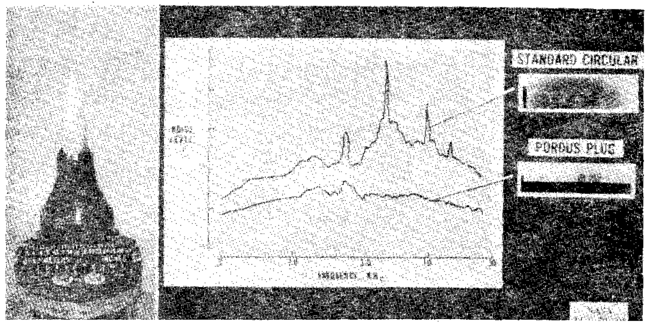
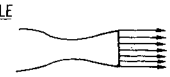



Fig. 23 Noise characteristics of a porous plug nozzle and a standard circular nozzle at a nozzle pressure ratio of 3.04 (after Maestrello[76]).

- APPROACH: ELIMINATE SHOCKS VIA ADJUSTABLE CONVERGENT-DIVERGENT NOZZLE



- DISADVANTAGES: EXTREMELY FORMIDABLE DESIGN PROBLEMS
- ADVANTAGES: SUBSTANTIAL THRUST GAIN AS WELL AS SHOCK NOISE SUPPRESSION
- CONCLUSION: COMPELLING ADVANTAGES EQUAL POWERFUL INCENTIVES

Fig. 24 Features of variable geometry convergent-divergent nozzle for suppression of shock noise.

Reduction of shock noise in supersonic jets is a matter of substantial current interest, along with predictive methods. The early theory of reference[70] accurately represents the physics, but for a very specialized scenario: homogeneous turbulence passing through an infinite plane shock. An adaptation has been made to accommodate the theory to the circumstances of the limited localized shocks in a jet.[74]

*Suppression*

The inverted velocity profile coannular jet—although disappointing on a PNdB basis for shock-free flow—appears to suppress shock structure, and, hence, shock noise, in some circumstances. The subject is under active experimental and theoretical study (e.g., Ref. 75).

Another approach of considerable interest is the *porous plug suppressor* of Maestrello,[76] shown in Fig. 23. The annular jet flows over a porous centerbody, which may be either sealed or vented to the atmosphere. Shadowgraphs show that the shock structure in the jet essentially is eliminated. Along with this, the shock noise is suppressed, as shown in the spectra on the figure.

The mechanism of the shock suppression is controversial. The porous surface + plug cavity cannot behave as a pressure-release surface; otherwise oblique shocks would reflect as expansion waves, only to reflect again from the jet boundary as shocks, and so on. It seems to this author that—somewhat like a slotted wall transonic wind tunnel—the porous surface may provide something on average intermediate between soft and hard boundary conditions. One could imagine alternating pores and rigid areas reflecting adjacent expansion and shock waves that mutually cancel. Not incompatible with this view, the porosity (ratio hole or pore area/total surface area) has been found to be an important performance parameter. The porous plug reduces the ordinary jet mixing noise as well as the shock noise, and appears to exact no thrust loss penalty. This makes it especially attractive.

*Convergent-Divergent Nozzle as Suppressor*

A properly contoured convergent-divergent nozzle operated at the design pressure ratio will emit a shock-free supersonic jet. Deviation from this pressure ratio will cause a proportional buildup of shocks, but they will be weaker than for a convergent nozzle. Thus a fixed convergent-divergent nozzle will eliminate shock noise at the design operating condition and reduce it at other conditions. It will also deliver more thrust than the convergent nozzle operated overexpanded.

A *variable* convergent-divergent nozzle (or equivalent axially translating variable centerbody) could yield shock-free flows up to its maximum design pressure ratio (Fig. 24). It would completely eliminate shock noise, and it would do this without thrust penalty. In fact, this is one of the rare schemes of jet-noise suppression that would provide a thrust bonus rather than a thrust loss as tradeoff.

A common view is that the design of a viable variable geometry convergent-divergent nozzle presents severe and uneconomic mechanical, weight, and durability problems. But the performance and noise advantages of such a design are extremely compelling: I would hope that they might provide sufficient motivation to accept the challenge. There is, of course, precedent in the engines of the F-15 and other supersonic fighter aircraft now flying.

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